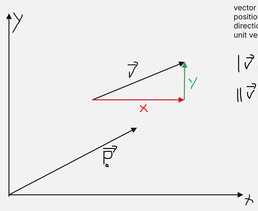
Vectors

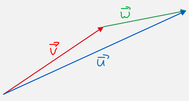


vector - parallel movement
position vector - cartesian point (vector from the origin)
direction vector - should always have a length of 1
unit vector - vector with a length of 1

$$|\vec{v}| = \sqrt{x^2 + y^2}$$

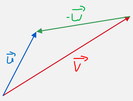
$$\|\vec{v}\| = \frac{\vec{v}}{|\vec{v}|}$$ normalized vector

Add

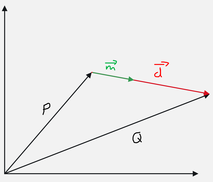


$$\vec{u} = \vec{v} + \vec{w} = \begin{pmatrix} x_v + x_w \\ y_v + y_w \\ z_v + y_w \end{pmatrix}$$

Sub



$$\vec{u} = \vec{v} - \vec{w} : \vec{v} + (-\vec{w})$$



Direction = Target minus Start

$$\vec{d} = \vec{Q_0} - \vec{P_0}$$ Distance Vector

$$\vec{m} = \|\vec{d}\| \cdot \left(s \cdot dt\right)$$ Movement Vector
s - Speed
dt - Delta Time

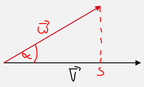
Multiplication with a Scalar



$$\vec{V} \cdot a = a \cdot \vec{V} = \begin{pmatrix} x \cdot a \\ y \cdot a \\ z \cdot a \end{pmatrix}$$

$$\vec{V} \cdot \left(s \cdot dt\right)$$ better performance

Dot Product - Scalar Product



Projection

$$\vec{v} \cdot \vec{w} = x_v \cdot x_w + y_v \cdot y_w + z_v \cdot z_w$$

$$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\alpha \quad \Leftrightarrow \quad \cos\alpha = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$$ $\cos^{-1}$ acos arccos

only use the arcus cosines/sines if you need the angle value for a calculation

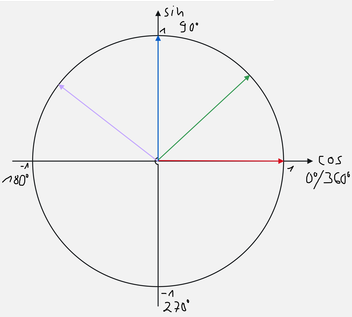
$$\alpha < \beta \rightarrow \cos\alpha > \cos\beta$$

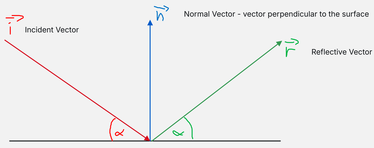
Radians vs. Degrees

$$\frac{\alpha}{180°} = \frac{x}{\pi}$$

$$\alpha = x \cdot \frac{180°}{\pi}$$ Rad2Deg

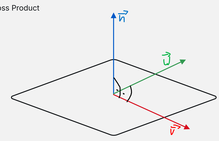
$$x = \alpha \cdot \frac{\pi}{180°}$$ Deg2Rad





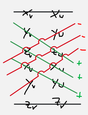
$$\vec{r} = \vec{i} + 2 \cdot (\vec{i} \circ \vec{n}) \cdot \vec{n}$$

Cross Product



$$\vec{n} = \vec{v} \times \vec{w} \qquad -\vec{n} = \vec{w} \times \vec{v}$$

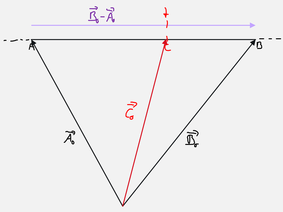
$$\vec{v} \times \vec{w} \neq \vec{w} \times \vec{v}$$



$$\vec{n} = \begin{pmatrix} y_v z_w - z_v y_w \\ z_v x_w - x_v z_w \\ x_v y_w - y_v x_w \end{pmatrix}$$

Interpolations

linear interpolation (lerp)



$$0 \leq t \leq 1$$

$$\vec{C_0} = \vec{A_0} + t \cdot (\vec{B_0} - \vec{A_0})$$

t - percentage on the way between A & B

linear extrapolation (lerpUnclamped)
PingPong

Bezier Curves

first degree - linear



$$B_1(t) = P_0 + t(P_1 - P_0)$$

$$= P_0 + tP_1 - tP_0$$

$$= (1-t)P_0 + tP_1$$

second degree - quadratic

$$B_0^2(t) = \boxed{(1-t)\left(B_{P_0P_1}(t)\right)} + t\boxed{B_{P_1P_2}(t)}$$

reuse already existing functions

$$= (1-t)^2 P_0 + 2(1-t)P_1 + t^2 P_2$$

performant version



third degree - cubic

$$B_C(t) = \boxed{(1-t)\left(B_{P_0P_1P_2}(t)\right)} + t\boxed{B_{P_1P_2P_3}(t)}$$

reuse already existing functions

$$= (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3$$

performant version

